Free Fall and Vertical Motion at Earth's Surface

Free fall describes the situation in which the only force acting on a falling object is gravity. In other words, a situation in which air resistance can be ignored. In such an idealized situation, all objects will have the same constant rate of acceleration:

$$g = 9.8\ m/s^2\ [down]$$

Solving Free Fall Problems

Problems involving freely falling bodies can be solved using the equations for constant acceleration:

$$d = \left(\frac{v_f + v_i}{2}\right) \cdot t \qquad\qquad v_f = v_i + at$$

$$d = v_i t + \frac{1}{2}at^2 \qquad\qquad v_f^2 = v_i^2 + 2ad$$

where the acceleration a is equal to the acceleration due to gravity g.

Example 1 Dropping an Object.
Suppose that a ball is dropped from a tower $70.0\ m$ high. How far will the ball have fallen after a time $t = 3.00\ s$?

Example 2 Throwing an Object Downwards.
Suppose you were in the observation deck of the CN Tower at a height of $342\ m$, and you threw a marble straight downwards with a velocity of $35.0\ m/s$. Assume there is no air resistance.

a) Determine the velocity of the marble just before it strikes the ground.

b) Determine the time it takes for the marble to reach the ground.

Example 3 Throwing an Object Upwards.
If you threw a ball straight upwards with a velocity of $25.0\ m/s$,

a) how long will it take to reach its maximum height?

b) how high would it go?

c) how long would it take to return to your hand?

Terminal Velocity and Air Resistance

In general, whenever something moves through the air, the air exerts a force on the object to oppose its motion — we call this force **air resistance** or **drag force**. The size of the drag force depends on two factors:

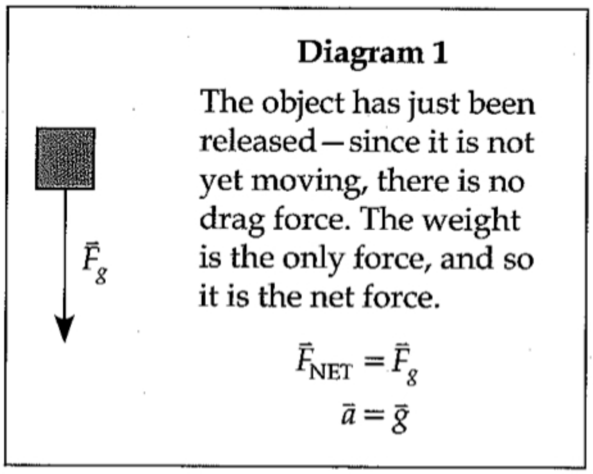
1. The speed of the object.
    - a faster moving object will experience a larger drag force

2. The surface area of the object.
    - objects with a larger surface area will experience a larger drag force

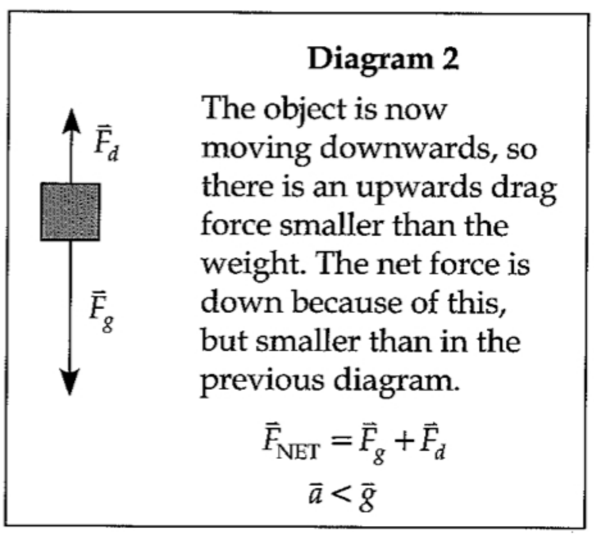
Terminal Velocity

Consider a falling object.

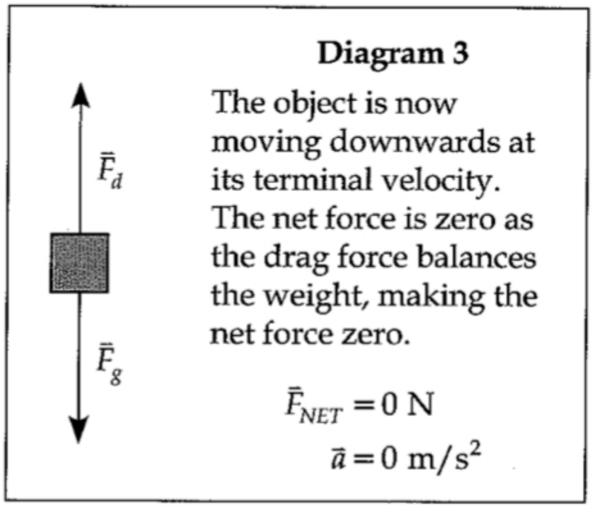
The net force causing the acceleration of the object is made up of two forces: the downward force of gravity (F_g), and the upward drag force (F_d).



As long as $F_g > F_d$, the net force will be downwards, and so downward acceleration will occur.



As the drag force increases (due to the object speeding up) the magnitude of the net force will decrease, thus decreasing the acceleration.



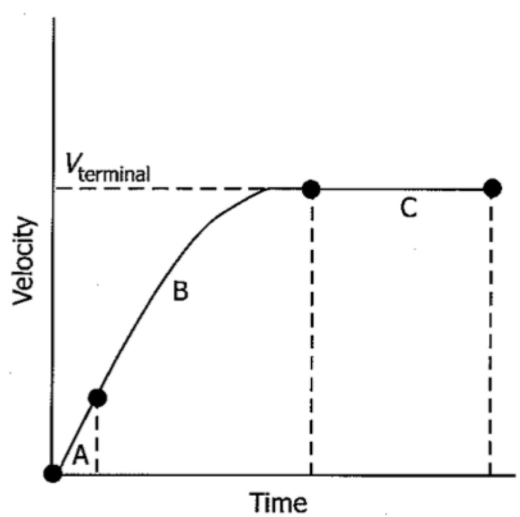
Eventually, the force of air resistance will equal the force of gravity. At this point, the net force on the object will be zero, and the object will no longer be accelerating (though it is certainly still moving!).

We say that the object has reached **terminal velocity**. The terminal velocity of an object is the velocity at which the drag force due to air resistance is equal in magnitude to the force of gravity. The net force on the object is zero, and its acceleration is zero.

The value for terminal velocity will vary, but in general it will be smaller for objects that are light compared to their size.

Terminal Velocity and a Parachutist

When a parachutist jumps from an airplane, a terminal velocity will be reached. The graph below shows possible velocities at various times (down is being used as the positive direction).



In section A, air resistance is negligible (small enough to be ignored), making gravity the only force worth considering. During this stage, the parachutist falls with an acceleration of $9.8\ m/s^2$ as we would expect. This results in a constant slope on the velocity-time graph.

In section B, the air resistance is no longer negligible, and is now increasing due to the increasing speed of the parachutist. This results in a decreasing net force and a decreasing acceleration. Thus, the graph is curved in this section, as the slope (acceleration) decreases. Note, however, that velocity continues to increase.

In section C, the air resistance has increased to the point where $F_g = F_d$ and the net force on the parachutist is zero. No acceleration occurs, so the slope of the velocity-time graph in this section is zero. The parachutist falls at a constant velocity — this is the terminal velocity. A parachutist falling with a closed parachute has a pretty high terminal velocity — approximately $200\ km/h$.

Once the parachute is opened, the falling person becomes a much larger object. Gravity does not change, but the air resistance becomes much larger. This means the upward drag force will be larger than gravity, producing an upwards acceleration, so the parachutist slows down.

The reverse of the above then happens: as the parachutist slows down, the drag force decreases until it again matches the force of gravity. A new, much smaller, terminal velocity is reached — approximately $20\ km/h$ — that is slow enough for a safe landing.

"Relax. My free-fall software shows we have a full twelve seconds until ripcord time."

Projectiles Worksheet #1

1. A stone is dropped from a bridge and strikes the water $2.5\ s$ later.
   a) What is the velocity of the stone just before it strikes the water? ($-24.5\ m/s$)
   b) How high is the bridge? ($30.6\ m$)

2. A ball is thrown downward with an initial speed of $20\ m/s$.
   a) What is the ball's velocity after $2\ s$? ($-39.6\ m/s$)
   b) How far does it fall in $5\ s$? ($222.5\ m$)

3. A ball is thrown upward with an initial speed of $20\ m/s$.
   a) What is the ball's velocity after $0.5\ s$? ($+15.1\ m/s$)
   b) What is its position after $3\ s$? ($+15.9\ m$)

4. Kyle is flying a helicopter when he drops a bag. When the bag has fallen for $2.0\ s$,
   a) what is the bag's velocity? ($-19.6\ m/s$)
   b) how far has the bag fallen? ($19.6\ m$)

5. Kyle is flying the same helicopter and it is rising at $5.0\ m/s$ when he releases the bag. After $2.0\ s$,
   a) what is the bag's velocity? ($-14.6\ m/s$)
   b) how far has the bag fallen? ($9.6\ m$)
   c) how far below the helicopter is the bag? ($19.6\ m$)

6. Kyle's helicopter now descends at $5.0\ m/s$ as he releases the bag. After $2.0\ s$,
   a) what is the bag's velocity? ($-24.6\ m/s$)
   b) how far has the bag fallen? ($29.6\ m$)
   c) how far below the helicopter is the bag? ($19.6\ m$)

7. A ball is dropped from the roof of a building $122.5\ m$ tall. Determine:
   a) the time taken for the ball to reach the ground. ($5\ s$)
   b) the velocity with which the ball strikes the ground. ($-49\ m/s$)
   c) the position of the ball at $t = 3\ s$. ($+78.4\ m$)
   d) the velocity of the ball at $t = 3\ s$. ($-29.4\ m/s$)

8. A ball is thrown straight down from the roof of a building $122.5\ m$ high with a speed of $10\ m/s$. Determine:
   a) the time of flight for the ball. ($4.1\ s$)
   b) the velocity with which the ball reaches ground level. ($-50.2\ m/s$)
   c) the position of the ball at $t = 2\ s$. ($+82.9\ m$)
   d) the velocity of the ball at $t = 2\ s$. ($-29.6\ m/s$)

9. A ball is thrown straight up from the top of a building $122.5\,m$ with an initial velocity of $+10\,m/s$. Determine:
   a) the time the ball is in free flight if it falls to ground level at the base of the building. ($6.12\,s$)
   b) the speed with which the ball reaches the ground. ($-50.0\,m/s$)
   c) the maximum height to which the ball rises. ($+5.10\,m$)
   d) the acceleration of the ball at its maximum height. ($-9.8\,m/s^2$)
   e) the position of the ball at $t = 0.5\,s$. ($+3.78\,m$)
   f) the velocity of the ball at $t = 0.5\,s$. ($+5.1\,m/s$)
   g) the position of the ball at $t = 3\,s$. ($-14.1\,m$)
   h) the velocity of the ball at $t = 3\,s$. ($-19.4\,m/s$)

"Oh, and I suppose it was me who said 'what harm could it be to give the chickens a book on nuclear physics?'"